Embedding Graphs Containing K_5-subdivisions

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By Kuratowski's theorem, a non-planar graph G contains a subdivision of K_5 or $K_{3,3}$ as a subgraph. Given a K_5-subdivision TK_5 in G, either the subdivision TK_5 can be transformed into a $K_{3,3}$-subdivision, or else G can be decomposed into smaller pieces according to the edges of K_5. We show that it is possible to decide if the graph G is projective-planar or toroidal by checking planarity of the decomposing pieces. As a result, we obtain practical linear-time algorithms to reduce a projective-planarity and/or toroidality testing to planarity checks or to a $K_{3,3}$-subdivision in the graph (joint work with W.L. Kocay).