How to detect a projective planar or toroidal graph

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We consider 2-cell embeddings of a graph in the projective plane and/or torus. Given a non-trivial 2-connected planar graph, an $O(1)$-time method is described for transforming a planar embedding into a 2-cell embedding in the projective plane or torus.

By Kuratowski's theorem, a non-planar graph G contains a subdivision of K_5 or $K_{3,3}$. Suppose that G contains a subdivision of K_5. We describe a linear-time algorithm which either reduces a projective-planarity-checking or toroidality-checking algorithm to a small constant number of planarity tests, or finds a subdivision of $K_{3,3}$ in G. The method is based on the structural properties of a K_5-subdivision and its embeddings in the projective plane and torus, and is easy to implement using a breadth-first or depth-first search.

We investigate the structure of the unique embedding of a $K_{3,3}$-subdivision in the projective plane. The structure of the embedding combined with the ideas of the Hopcroft-Tarjan planarity algorithm gives a projective-planarity algorithm for graphs with a $K_{3,3}$-subdivision. As a result, we have a practical linear-time algorithm for determining whether a graph is projective-planar.